From this last form he deduces the linear equation for r,

$$r = \frac{h^2}{m} + \lambda.x + \gamma.y,$$

h, λ and γ being constants. In his Theoria Motus, art. 3, Gauss uses this equation in the form

$$r + \alpha.x + \beta.y = \gamma,$$

and has the remark "et quidem γ quantitatem natura sua semper positivam;" which from Laplace's form is evidently correct.

If we integrate equation (1) we have

$$\frac{d^2 r}{d t^2} + \frac{m}{r^2} + \frac{k}{r^3} = 0,$$

the constant k having a value different from zero. Hesse gives the following interpretation to this equation: If one lived on the radius vector of a planet he would not be able to explain the motion of the planet by means of the law of gravitation, but must regard the inverse third power of the distance.

TWELVE ORIGINAL PROBLEMS.

BY PLINY EARLE CHASE, LL. D., PROF. OF PHILOS. IN HAVERFORD COL.

1. LET f represent any central force whatever, varying inversely as the square of the distance, r being the radius of a perpetual circular oscillation produced by the force. Required the mean velocity of a synchronous radial oscillation.

2. If material particles under the influence of f were constrained to move in various orbits, (circular, elliptic, parabolic, and rectilinear,) collisions near the centre would produce a nucleus, which would rotate, on account of the resultant of such portions of the orbital forces as were not otherwise represented. Required the law of varying velocity, in terms of r, under nucleal contraction or expansion.

3. If a viscous nucleus were accompanied by a non-viscous, elastic, atmosphere, rotating with the nucleus on account of central pressure, what formula would express the limit of possible atmosphere, in terms of f, r, and v, r being the nucleal radius and v the velocity of equatorial rotation?

4. If such a nucleus and atmosphere are simultaneously expanding or contracting, express the varying ratio of the nucleal radius (r) in terms of the atmospheric radius (r').

5. Give an expression for the common tangential velocity towards or from which the circular-orbital velocity (v') and the equatorial-rotation velocity (v) both tend.

6. Give an expression for the common tangential velocity, towards which the parabolic-perifocal, or dissociating velocity ($v'\sqrt{2}$) and the equatorial-rotation velocity both tend.

7. Give an expression for the common velocity towards which the dissociating velocity ($v'\sqrt{2}$) and the mean radial velocity (Prob. 1) both tend.

8. Required the values of the common velocities in the Solar System, (probs. 5, 6, 7,) estimating the time of solar rotation at 25.4 days.

9. Required the value of the ultimate velocity (Prob. 7) for the largest planets in the intra-asteroidal and the extra-asteroidal belt (the Earth and Jupiter), estimating Jupiter's day at 9.6 h.

10. Granting the postulates of Problems 8 and 9, what is the sun's mean distance from the earth?

11. What would be the ratio of elasticity to density, in any medium which would admit of the least velocity assigned to gravity by Laplace, (100,000,000 times the velocity of light), the ratio in air being assumed as unity?

12. What must be the nature of a medium which would admit of an instantaneous velocity, such as Laplace supposed the velocity of gravity to be?—[To the foregoing probs. Prof. Chase appends the following answers.]

1. $t = 2\pi\sqrt{(f \div r)}$; $\therefore 4\pi r = 2t\sqrt{(fr)}$. 2. The conservation of areas requires that $v \propto (1 \div r)$. 3. $r\sqrt{(fr)} \div v$. 4. $r \propto \sqrt{t}$; $r' \propto \sqrt[3]{t^2}$; $\therefore r' \propto \sqrt[3]{r^4}$. 5. $v' \propto \sqrt{(1 \div r)}$ $\therefore (fr \div v^2) \times v = fr \div v$. 6. $2fr \div v$. 7. $(2v \div \pi)(\pi^2 \div 4) = \frac{1}{2}\pi v$; substituting for v its limit, $2fr \div v$, we get $\pi fr \div v$. 8. t of planetary revolution at sun's surface $= 1$ year $\div \sqrt{(214.86)^3} =$ 10,020s. $v' = \sqrt{fr} = 2\pi r \div 10{,}020$; $v = 2\pi r \div (25.4 \times 86400)$. Therefore Prob. (5) $= .13734r$; (6) $= .27468r$; (7) $= .4316r = v.$ *of light*; for $v.$ of light $= 214.86r \div 497.825 = .4316r$. Sun-spot observatons give rotation-periods varying between 24.6 days and 25.5 days. 9. $v' = \sqrt{(3963 \times 5280 \times 32.08)} \div 5280 = 4.9$m.; $v = 24890 \div 86164 = .289$m.; $\pi fr \div v = 261$m. $= v.$ of planetary revolution *at mean c. g. of Sun and Jupiter.* Herschel's estimate for Jupiter's day is about 4 per cent. greater than that assumed in the problem. 10. Jupiter's dist. $\div$ by sum of masses $= 5.2028 \times 214.86 \div 1048.879 = 1.0658$; $\sqrt{(1.0658)} \times 261 \times 10020 \times 214.86 \div 2\pi = 92{,}115{,}000$m. 11. Estimating $v.$ of sound at .216m.; $(185{,}034 \times 100{,}000{,}000 \div .216)^2 = 7{,}338{,}321{,}000{,}000{,}000{,}000{,}000{,}000{,}000$. 12. It can have no inertia, and cannot, therefore, be a material medium.

NOTE, BY ALEX. EVANS.—I find that in the last edition, 1876, Sec. 514 of the Outlines of Astronomy by Sir J. F. W. Herschel, published by Appletons, N. Y., the period of Saturn's rotation is restored to $10^h\ 16^m\ 00.44^s$.